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1. [14.3.90] **The heat equation** An important partial differential equation that describes the distribution of heat in a region at time t can be represented by the *one-dimensional heat equation*

$$\frac{\partial f}{\partial t} = \frac{\partial^2 f}{\partial x^2}.$$

Show that $u(x,t) = \sin(\alpha x) \cdot e^{-\beta t}$ satisfies the heat equation for constants α and β. What is the relationship between α and β for this function to be a solution?

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2. [14.6.54] **The Wilson lot size formula** The Wilson lot size formula in economics says that the most economical quantity Q of goods (radios, shoes, brooms, whatever) for a store to order is given by the formula $Q = \sqrt{2KM/h}$, where K is the cost of placing the order, M is the number of items sold per week, and h is the weekly holding cost for each item (cost of space, utilities, security, and so on). To which of the variables K, M, and h is Q most sensitive near the point $(K_0.M_0, h_0) = (2, 20, 0.05)$? Give reasons for your answer.

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3. [14.7.68] **Least squares and regression lines** When we try to fit a line $y = mx + b$ to a set of numerical data points $(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$, we usually choose the line that minimizes the sum of the squares of the vertical distances from the points to the line. In theory, this means finding the values of m and b that minimize the value of the function

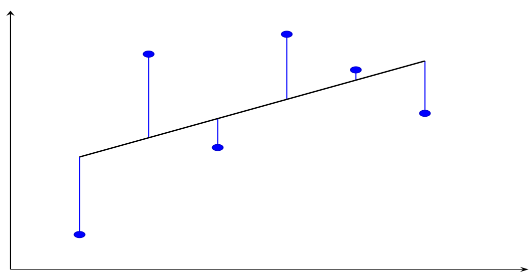
$$w = (mx_1 + b - y_1)^2 + \cdots + (mx_n + b - y_n)^2. \tag{1}$$

The values of m and b that do this are

$$m = \frac{\left(\sum x_k\right)\left(\sum y_k\right) - n\sum x_k y_k}{\left(\sum x_k\right)^2 - n\sum {x_k}^2} \tag{2}$$

$$b = \frac{1}{n}\left(\sum y_k - m\sum x_k\right), \tag{3}$$

with all sums running from $k = 1$ to $k = n$. Many scientific calculators have these formulas built in, enabling you to find m and b with only a few keystrokes after you have entered the data.



The line $y = mx + b$ determined by these values of m and b is called the *least squares line*, *regression line*, or *trend line* for the data under study. Finding a least squares line lets you

- summarize data with a simple expression,
- predict values of y for other, experimentally untried values of x, and
- handle data analytically.

Use equations (2) and (3) to find the least squares line for the set of data points: $(0, 0), (1, 2), (2, 3)$. Then use the linear equation you obtain to predict the values of y that would correspond to $x = 4$.

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4. [14.Prac.85] **Minimizing cost of a box** A closed rectangular box is to have volume V cm^3. The cost of the material used in the box is a cents/cm^2 for top and bottom, b cents/cm^2 for front and back and c cents/cm^2 for the remaining sides. What dimensions minimize the total cost of materials?

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5. [14.Adv.13] **Minimum volume cut from first octant** Find the minimum volume for a region bounded by the planes $x = 0$, $y = 0$, $z = 0$ and a plane tangent to the ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

at a point in the first octant.